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ANALYSIS AND DESIGN OF LEVITATION BASED VIBRATION ENERGY HARVESTERS

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Abstract. A computational approach for the evaluation of the electro-mechanical response of levitation based vibration energy harvesters is presented in this paper. The key aspects of the design of levitation based energy harvesters, such as the existence of the resonance phenomenon, the influence of damping in the system response, the magnetic force nonlinearity and the calculation of the magnetic flux derivative for multi-magnet configurations are addressed. The evolution in time of the electromechanical variables is investigated through a hybrid numerical-analytical approach. The evaluation of the levitational force and the magnetic flux derivative is done through a nonlinear model based on the finite element method. A performance assessment is done by comparing the results obtained with the present formulation against measurements; a physical prototype of a multi-pole-multi-coil harvester is built ad hoc. An excellent agreement between the mathematical model and the experiments was found.

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1 INTRODUCTION

Levitation based electromagnetic energy harvesters are an interesting alternative for scavenging energy from vibration sources, especially for low frequency applications; the simplicity of its construction together with its low maintenance are points that justify this fact. Investigations about levitation based harvesters are not abundant; a few works can be found in the literature (Mann and Sims, 2009; Zhu and Zu, 2012; De Pasquale, Iamoni and Somà, 2013; Apo and Priya, 2014; Palagummi, Zou and Yuan, 2015; Soares Dos Santos *et al.*, 2016). Most of this works are based on an analytical formulation for the magnetic field, which implies that the magnet geometry must be simplified. Several important aspects such as the optimal design of multipole configuration, the accurate modeling of the levitation force, the analysis of the nonlinear response, etc. have still not been addressed.

Zhu and Zu (Zhu and Zu, 2012) presented the simulation of a vibration-based energy harvester that uses a magneto-electric laminate composite to harvest energy from the nonlinear vibrations of a levitating magnet. A cubic law was used to model the levitation force and an average axial flux analytical formula was used to calculate the axial magnetic flux. Abed et al. (Abed *et al.*, 2015) studied the non-linear dynamics of a two degrees of freedom levitation based harvester. The magnetic forces were calculated with simplified analytical expressions; the bandwidth enhancement possibilities of the 2 DOF system was studied. It is not stated in the paper how the magnetic flux was obtained. Mann and Sims (Mann and Sims, 2009) have investigated the design and analysis of a novel energy harvesting device that operates as a tunable oscillator. The researchers proposed a cubic polynomial law for modeling the magnetic force, the coefficients of the polynomial were obtained experimentally. The electromechanical coupling was modeled through a damping coefficient, also obtained experimentally. The response of the system for harmonic excitation was obtained analytically through the method of multiple scales; the dynamics of the system was compared with experiments. No information about the magnetic field distribution and power curves was presented.

Soares dos Santos et al. (Soares Dos Santos *et al.*, 2016) have reported a levitation based harvester with a single Neodymium moving magnet. The authors developed a semi-analytical approach based on a surface current model for calculating the magnetic flux distribution; the equation of motion were integrated numerically. The force between the magnets was obtained through derivative of their interaction energy; this requires the assumption that the magnets are coaxial. Friction between the magnet and the cylinder was considered through a Karnopp model. Some sort of finite rotation framework in terms of Euler angles was presented to account for container arbitrary dynamics; although, the non-commutativity of rotations was not addressed. Lee et al. (Lee *et al.*, 2010) also presented a levitation based harvester composed of a single magnet moving inside a coil. Both cubic and quintic polynomials were used to model the magnetic force between the moving magnet and the end magnets. The coefficients of the polynomial were obtained experimentally. The electromechanical coupling was also obtained through experiments. The magnetic flux was assumed to be uniform and constant in time; thus the variation of the magnetic field with the displacement of the moving magnet was not considered. Munaz et al. (Munaz, Lee and Chung, 2013) studied different design architectures of an electromagnetic harvester. The magnetic flux density was modeled with analytical functions; the superposition principle was recalled when multipole magnets were studied. A finite element analysis was conducted to find the optimal magnetic flux distribution according to the number of poles of the moving magnet. Experimental analyses were conducted and compared against the power calculated assuming that the magnet moves according a sinusoidal law; so, the dynamic of the system was imposed and the

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electromechanical coupling was neglected. The coil configuration used to avoid cancellation in the multipole configuration was not informed.

Apo and Priya (Apo and Priya, 2014) studied the effect of multipole configurations for the moving magnet of a levitation based harvester. The levitating force equation was found to be a cubic polynomial. Finite element analysis was conducted to find optimal magnet configuration. No details about the solution of the equations of motion were given. Dallago et al. (Dallago, Marchesi and Venchi, 2010) developed an analytical model for considering the nonlinear stiffness effect on levitation based harvesters. A cubic law for the magnetic force was used; the polynomial coefficients were found through FEM. Avila Bernal and García (Avila Bernal and Linares García, 2012) presented a mathematical derivation for modeling the dynamics of mono-pole electromagnetic energy harvesters. Analytical models for the magnetic field distribution and magnetic force were presented. The analytical results were compared against FEM.

A different class of levitation based harvesters are diamagnetic levitation harvesters; these devices completely avoid the use of containers and therefore friction is eliminated from the problem. In this direction, Wang et al. (Wang *et al.*, 2013) and Palagummi et al. (Palagummi, Zou and Yuan, 2015) have developed a harvester with dual diamagnetic plates and a cylindrical levitating magnet. Some important conclusions can be made analyzing the cited works; the magnetic flux was modeled with analytical expressions or even not modeled at all. This is a important fact since the correct modeling of the magnetic flux density is crucial to obtain an accurate prediction of the generated power and also of the levitation force. Some works claim the analytical modeling of the magnetic flux is superior to numerical modeling in terms of accuracy and computational cost (Soares Dos Santos *et al.*, 2016). Although it could be possible to justify the contrary, it suffices to say that analytical approaches only can deal with very simplified geometries. Note that still the involved formulations used by Santos et. al (Soares Dos Santos *et al.*, 2016) and Avila Bernal and García (Avila Bernal and Linares García, 2012) to model the axial magnetic flux distribution cannot account for the effect of the flux interference caused by the end magnets nor the effect of spacers in multipole magnets. Accurate analytical modeling of the magnetic flux distribution for levitating harvesters is possible only for simple magnet geometries.

This work presents a general mixed formulation for modeling linear electromagnetic energy harvesters. The work is focused on the development of a computational procedure to predict the time variation of the induced voltage in a complex energy harvester. The approach is based on a hybrid formulation that is capable of dealing the most general case of linear harvester configuration. The evolution of the mechanical variables is obtained through an analytical model while the magnetic variables are obtained through the finite element method. Both models are linked through a series of computational routines that involve: polynomial fitting, numerical integration and data lookup algorithms.

The magnetic flux density is calculated using finite elements; a computational algorithm extracts the average flux as a function of the axial coordinate and calculates its derivative via a central difference scheme. The levitation force at discrete locations is also obtained with the same finite element data, thus avoiding the use of analytical expressions which are limited to single-pole magnets. This also avoids the use of experimentation to obtain the force law. A polynomial fitting technique is used to finally obtain an analytical force-displacement function. The equations of motion of the electromechanical system are written as a function of the average flux derivative; then they are transformed to the state space in order to be solved through a numerical integration algorithm.The approach is capable of modeling devices with arbitrary magnet geometry, arbitrary magnet pole and coil counts, and random mechanical excitations. Several nonlinear affects, such as:

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frictional, viscous and electromagnetic damping are naturally included in the model. The proposed approach has great flexibility and generality of a full tridimensional coupled electromechanical simulation and still retains the simplicity of one dimensional approaches. The validation of the formulation is done through a detailed comparison against experimental data obtained from physical testing of a prototype multi-pole multi-coil harvester built ad hoc. Also, some key aspects of levitation based harvesters, such as the jump phenomenon and its sensitivity to the system friction and load amplitude are briefly addressed.

2 DESIGN GENERALITIES

All Levitation Based Vibrational Energy Harvesters (LBVHs) have a similar architecture; a magnet or stack of magnets moves inside a cylinder that is surrounded by a coil. The motion of the magnet is limited by a repulsive force exerted by the opposing magnetic field of auxiliary magnets placed in the cylinder ends. In this work, we propose a multi-coil-multi-magnet-multi-spacer configuration of a LBVEH, see Figure 1. The objective of this design is to generate a complex voltage-time signal and then test the proposed approach in this complex scenario. In order to simplify the language, the central magnet assembly of magnets and spacers will be called “stack”. The design of the harvester starts with the study of the behavior of the dynamical system; this requires to find the equations that describe the force interaction between the components. The repulsive force exerted by the auxiliary end magnets is nonlinear with the distance between them and the moving magnet; thus, they play the role of a nonlinear stiffness. The magnets repulsive force law can be reasonably approximated with a polynomial (Dallago, Marchesi and Venchi, 2010; Zhu and Zu, 2012; Pasquale, Somà and Fraccarollo, 2013; Apo and Priya, 2014); since it is the responsible of the stability of the dynamical system, odd power functions are required.

A LBVEH is designed to recover energy from vibration sources, the nature of the source constraints the harvester tuning. There is a common misinterpretation in most research works, the harvester is said to be designed to have a resonant frequency that is coincident with the predominant frequency of the source (Saha *et al.*, 2008; Dallago, Marchesi and Venchi, 2010; Zhu and Zu, 2012; Munaz, Lee and Chung, 2013; Apo and Priya, 2014). This is not strictly correct since an LBVEH do not have a resonant frequency; it is known the fact that an oscillator with nonlinear stiffness can be modeled by the Duffing’s equation, which do not exhibit resonance (Griffiths and Inglefield, 2005).

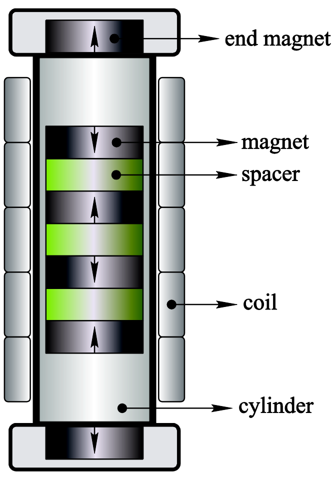

Figure 1 - Levitation-based harvester architecture

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2.1 Electro-mechanics

As the stack of magnets move inside the cylinder, the magnetic flux through the coils changes with time; inducing an electric field in the coil. This electric field generates an electro-motive force $\mathcal{E}$; which is equal to:

$$\mathcal{E} = \oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt}\int_S \boldsymbol{B} \cdot \boldsymbol{n}\, dA = -\frac{d\varphi}{dt} \tag{1}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic flux density, C is the curve described by the coil, S is the surface enclosed by the coil loops, $\boldsymbol{n}$ is the surface unit normal vector and φ_m is the magnetic flux through S. So, the total voltage induced in a coil loop is obtained through the integral of the electric field over the curve described by the loop; using Faraday's law this results to be equal to the time derivative of the total magnetic flux through the surface enclosed by the loop (Griffiths, 2010).

Since the stack is contained by the cylinder, it can only move in the axial direction *x*; the loops enclosed surfaces have a unit normal vector that is coincident with the axial direction. Therefore, the problem is uniaxial and the magnetic flux is only a function of the axial coordinate. Then the induced voltage can be written as

$$V = -\frac{d\varphi_m}{dx}\frac{dx}{dt} = -\varphi'\dot{x} \tag{2}$$

where the overdot $(\dot{\ })$ indicates time derivative and the accent superscript indicates derivative respect to x.

The discrete evaluation of the magnetic flux derivative is possible, so

$$V = -\left(\sum_{i=1}^{nc}\int_S B'(x_i)\, dA_i\right)\dot{x} = -\sum_{i=1}^{nc}\varphi_i'\,\dot{x} \tag{3}$$

where A_i is the area of the *ith* coil and B_i' is the x derivative of the x component of the magnetic flux density and *nc* is the total number of coils. The last equation clearly shows that in order to generate maximum voltage both the magnetic flux derivative and velocity of the stack must be optimized.

Now, the equations of motion of the mechanical system can be derived from Newton's law, so

$$F(x, \dot{x}, \varphi_m, t) = m\ddot{x}_s \tag{4}$$

where F is the sum of all the forces acting on the stack, m is the stack mass and $\ddot{x}_s$ is the stack acceleration. The forces acting on the system are:

$$F = F_k(x, \varphi_m) + F_c(\dot{x}, \varphi_m') + F_g \tag{5}$$

where F_k is the levitation force, F_c is the damping force and F_g is the gravitational force.

2.2 Magnetic levitation force

The restoring forces acting on the stack are exerted by the end magnets; as the stack moves into the cylinder the end magnets play the role of a spring. The opposing magnetic fields of the end magnets and the stack generates a stabilizing force that is function of the distance between them. In the present work, Finite Element modeling was used to obtain

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the force-displacement law of the harvester; the computer code FEMM was used to perform the simulations (Meeker, no date). We found that for the present configuration the magnetic force can be approximated with the following function of the stack displacement

$$F_k = a_0^k u_s + a_3^k u_s^3 + a_5^k u_s^5 \quad (6)$$

where a_i^k are constant parameters and u_s is the displacement of the stack relative to the cylinder base. To obtain a centered function, the initial position of stack was taken at the middle point of the cylinder length; then, the relative displacement of the stack can be written as

$$u_s = x_s - x_b - l_{sb} \quad (7)$$

where x_s is the stack position, x_b is the base position and l_{sb} is the initial magnet-base distance. Note this choice implies that the gravitational load must be imposed to the model.

2.3 Damping forces

There are three main sources of damping in a LBVEH: friction, air damping and electromagnetic damping. Air and electromagnetic damping are proportional the velocity, while friction damping is near constant with velocity.

The relative velocity of the stack can be obtained by derivation of Eq. (7) as

$$\dot{u}_s = \dot{x}_s - \dot{x}_b \quad (8)$$

The air damping is the result of the air flux through discharge holes made in the harvester to avoid internal air compression. The simplest, but yet effective, model that can be written to account for air damping is a proportional viscous model; therefore, the air damping force will be obtained as:

$$F_c^s = c_s \dot{u}_s, \quad (9)$$

where is the viscous damping coefficient that will be obtained by identification.

The frictional force can be obtained using a Coulomb model as

$$F_c^f = c_f \operatorname{sign}(\dot{u}_s) = \frac{c_f}{|\dot{u}_s|} \dot{u}_s. \quad (10)$$

To derive a unique damping coefficient, it is convenient to rewrite the above equation as

$$F_c^f = \frac{c_f}{|\dot{u}_s|} \dot{u}_s. \quad (11)$$

The induction damping force is generated when the circuit is closed with a load, and thus a current is created. This current creates its own magnetic field; this new magnetic field opposes to its cause, giving rise to a force that opposes the motion. Since this force is proportional to the current and thus proportional to the relative velocity of the stack, it is often considered as an electromagnetic damping force. So, it can be written as

$$F_c^m = c_m \dot{u}_s, \quad (12)$$

where c_m is the electromagnetic damping coefficient.

To derive the expression of the electromagnetic damping an energy balance law must be recalled. The electromagnetic damping force is responsible for the electric power

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generation, so the energy conversion must be such that the mechanical power dissipated by the electromagnetic damping is equal to the generated electrical power, i.e.

$$F_c^m \dot{u}_s = \frac{V^2}{R_l + R_c + j\omega L_c}, \tag{13}$$

where V is the induced voltage, R_l is the load resistance, R_c is the coil resistance, L_c is the coil inductance and F_c^m is the electromagnetic damping force.

Using Eqs. (3) and (12), the above equations give the electromagnetic damping coefficient as

$$c_m = \frac{(\sum \varphi_i')^2}{R_l + R_c + j\omega L_c} \tag{14}$$

At low frequencies the coil inductance has a negligible effect compared to that of the resistance, so the above equation can be written as:

$$c_m = \frac{(\sum \varphi_i')^2}{R_T}, \tag{15}$$

where R_T is the total resistance of the circuit.

2.4 Inductive performance

The maximization of the induced voltage implies the maximization of the product of the velocity and the magnetic flux derivative. So, the efficiency of an electromagnetic harvester is governed by two factors: its dynamics and its induction capability. The latter is dependent on the magnetic field gradient strength of the moving magnet, the number of coil loops count and loops area. An analytical law is commonly used to model the magnetic flux distribution along the axial and radial components (Mann and Sims, 2009; Lee *et al.*, 2010; Avila Bernal and Linares García, 2012; Zhu and Zu, 2012; Soares Dos Santos *et al.*, 2016). Sometimes this law is used for multi-magnet stacks, assuming additivity, without proving its validity.

The magnetic field gradient strength can be optimized for certain configurations of the stack by using ferromagnetic spacers; the geometrical design of both spacers and the magnets are of key importance to ensure a high flux derivative. The effect of ferromagnetic spacers on the magnetic flux derivative is remarkable, unfortunately it cannot be modeled by analytical approaches. The spacers allow the flux spatial derivative to grow and also change sign; it can be said that they work as backirons. The growth is such that if one replaces the spacers by magnets the flux derivative is considerably lower. The Figure 2 shows a comparison of the flux derivative for a four magnet stack and three spacers configuration vs. a seven magnet stack configuration.

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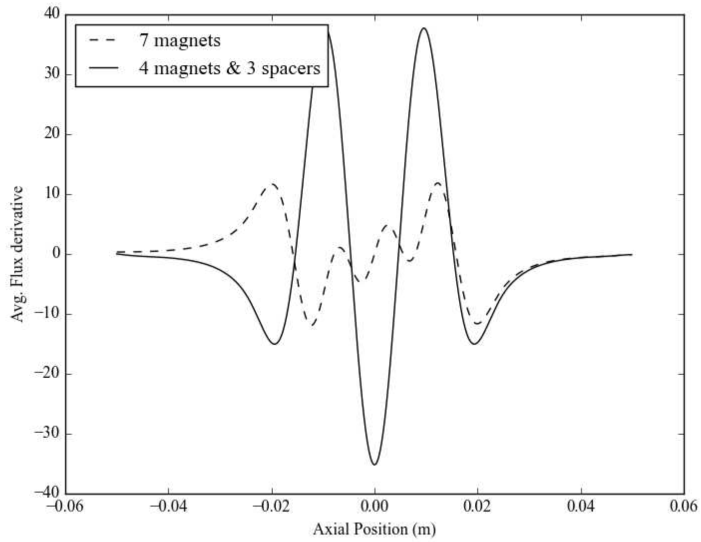

Figure 2 - Magnetic flux derivative, effect of spacers

The accurate evaluation of the magnetic field distribution is of paramount importance for the prediction of the harvester performance. The Finite Element Method is probably the best tool for that purpose; it is simple, fast and it can handle arbitrary geometries.

In order calculate the flux derivative at a certain location and time instant we propose an algorithmic approach that processes the magnetic finite element information and operates numerically on the result to obtain a vector of flux derivatives. This vector in conjunction with a 1-D interpolation algorithm to evaluate the flux derivative in every coil of the harvester in a certain time instant.

The algorithm has the following procedural structure:

I. Read the finite element results of the magnetic flux (Lua Script).
II. Calculate the average flux density over the coil area in the origin of the axial coordinate (Lua Script).
III. Store the coordinate and the average flux in an ASCII file (Lua Script).
IV. Advance a step in the coordinate and repeat I-III until the last coordinate length (Lua Scritp).
V. Read the average flux file (Python).
VI. Derivate the average flux vector using central difference (Python).
VII. Smooth the derivative using a Savitzky-Golay filter (Python).

There is an important point that must be considered for calculating the magnetic flux of a LBVEH; the end magnets perturb the flux in the stack. This perturbation may be not important if the stack is not moving significatively, but can be of importance if the stack moves close to the end magnets. This raises an important problem, since now the magnetic flux distribution is changing with the magnet displacement, and then with time. The Figure 3 shows how the magnetic flux derivate changes its shape as the stack moves closer to the end magnet.

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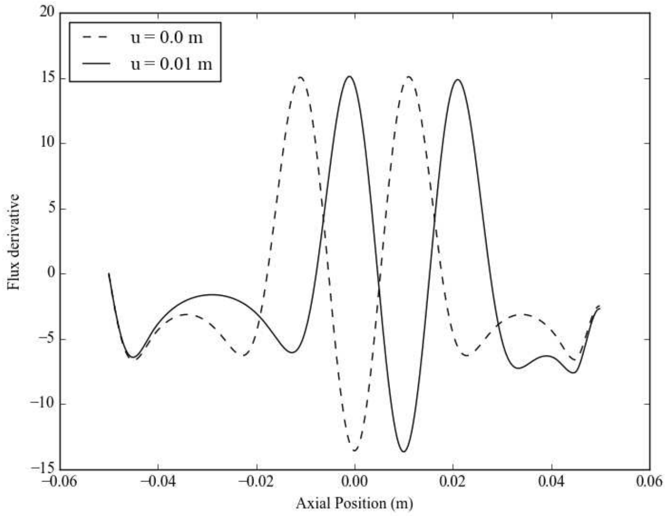

Figure 3 - Magnetic flux derivative

The variation of the magnetic flux pattern with the magnet displacement pose a question about the necessity of considering this variation in the determination of the generated voltage. From the computational point of view, considering the flux variation is cumbersome; two approaches could be used to tackle the problem: a full finite element analysis of the coupled problem or a so-called quasi-static determination of the flux for different locations of the magnet with a subsequent point searching technique to determine the flux after the solution of the dynamic problem. Fortunately, the change in the flux derivative is accompanied with a reduction in velocity of the stack; which implies that, still when the magnetic flux distribution is perturbed by the magnets close up, the effect on the voltage generation may not be important. For the sake of brevity, this effects will be disregarded in the present paper; although, it will be treated in a future work.

It is important to note that a multi-coil configuration requires an out of phase connection of the coils; otherwise the group of coils would behave as a large single coil and thus the term multi-coil would be incorrect.

2.5 Power optimization

The optimization of the harvester power generation capability implies the maximization of the integral of the product of the velocity and the magnetic flux derivative. The source vibration imposes the operating conditions of the harvester; therefore, its optimal design requires knowing the time and frequency characteristics of the source signal. The most simplified model of a LBVEH could be well represented by the Duffing's equation; the dynamics of the Duffing's equation is well known, and it could be well justified that, unlike linear systems, vibration amplitudes raise unlimitedly with frequency.

It is commonly accepted that an effective harvester should be tuned to work in a resonant condition (Saha *et al.*, 2008; Mann and Sims, 2009; Dallago, Marchesi and Venchi, 2010; Munaz, Lee and Chung, 2013; Apo and Priya, 2014; Abed *et al.*, 2015; Palagummi, Zou and Yuan, 2015). However, magnetic harvesters do not exhibit resonance; this assertion will be clarified later. This misinterpretation probably appears because of the fact that in the presence of damping, the amplitudes jump from a large displacement equilibrium path to a small displacement one, thus producing a resonance like response. The jump frequency is sensitive not only to damping, but also to the load

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amplitude. In virtue of that, it is very important to note that, unlike linear oscillators, the frequency tuning of a LBVEH is strongly dependent on the load amplitude and on the system damping.

A reasonable criterion to maximize the velocity of the stack of magnets relative to the coil is to maximize the stack stroke; which has the natural consequence of increasing size. If sinusoidal excitation is assumed, the harvester vibration frequency fixed, so maximizing the amplitude is the next step. At this point, the geometric dimensions of the device define the elasticity of the system. The harvester mass is fixed by the magnetic induction design through the harvester power requirements.

3 EQUATIONS OF MOTION

3.1 Mechanical system

The equation of motion are derived from the force balance in the stack; we will refer the displacements of the stack to the displacements of the cylinder; therefore, the resulting formulation can be considered to be *relative*. Note that an absolute formulation could also be derived, but it is clearly not advantageous in cases where base acceleration is used as excitation since the evaluation of the external forces is considerably more involved. In turn, if the base displacement is chosen as excitation, an absolute formulation is advised.

Recalling Eqs. (4) and (5), the equations of motion can be written as

$$F_k(x, \varphi_m) + F_c(\dot{x}, \varphi'_m) + F_g = m\ddot{x}_s, \tag{16}$$

being F_k, F_c and F_g the levitation, damping and gravitational forces respectively and $\ddot{x}_s$ the absolute acceleration of the stack.

Using Eqs. (6), (9), (12) and (15) the equations of motion can be expanded as

$$m\ddot{x}_s + a_0^k u_s + a_3^k u_s^3 + a_5^k u_s^5 + \left[c_s + \frac{c_f}{|\dot{u}_s|} + \frac{(\sum \varphi'_i)^2}{R_T}\right]\dot{u}_s + mg = 0 \tag{17}$$

The stack acceleration can be obtained by derivation of Eq. (7), so

$$\ddot{x}_s = \ddot{u}_s + \ddot{x}_b \tag{18}$$

Defining the nonlinear stiffness

$$k(u_s) = a_0^k + a_3^k u_s^2 + a_5^k u_s^4 \tag{19}$$

And the damping coefficient

$$c = c_s + \frac{c_f}{|\dot{u}_s|} + \frac{(\sum \varphi'_i)^2}{R_T} \tag{20}$$

The equations of motion can be written in the compact form

$$m\ddot{u}_s + k(u_s)u_s + c(\dot{u}_s, \varphi'_i)\dot{u}_s = -m(\ddot{x}_b + g) \tag{21}$$

From the last equation, it can be clearly seen that all unknowns are relative magnitudes.

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3.2 Electrical system

As already said, the change in magnetic flux induces a voltage in the coils. When the circuit is closed with a load, a current flows through the circuit. The equations of the electrical system can be written as

$$L\dot{I} + R_T I = V \tag{22}$$

where I is the current flowing through the circuit, L is the inductance of the coils, R_T is the total resistance of the circuit and V the induced voltage.

Using Eq. (3), the voltage can be expressed as a function of the stack velocity, then the electrical system equations take the form

$$L\dot{I} + R_T I = -\sum \varphi_i' \, \dot{u}_s \tag{23}$$

As it can be seen, this is an electro-mechanical equation, coupled through the derivative of the magnetic flux.

3.3 Coupled system

The electro-mechanical system is modeled by a coupled system of nonlinear equations. Bringing together the mechanical and the electrical equations, Eq. (21) and Eq. (23) respectively, we have

$$\begin{gathered} m\ddot{u}_s + k(u_s)u_s + c(\dot{u}_s, \varphi_i')\dot{u}_s = -m(\ddot{x}_b + g) \\ L\dot{I} + R_T I = -\varphi_i' \dot{u}_s \end{gathered} \tag{24}$$

In matrix form the above equations can be written as

$$\begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_s \\ \ddot{I} \end{bmatrix} + \begin{bmatrix} c_s + \dfrac{c_f}{|\dot{u}_s|} + \dfrac{\varphi_i'^2}{R_T} & 0 \\ \varphi_i' & L \end{bmatrix} \begin{bmatrix} \dot{u}_s \\ \dot{I} \end{bmatrix} + \begin{bmatrix} k(u_s) & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} u_s \\ I \end{bmatrix} = \begin{bmatrix} -m(\ddot{x}_b + g) \\ 0 \end{bmatrix} \tag{25}$$

In order to numerically integrate these equations using the Runge-Kutta algorithm the system must be recast in state space form, then

$$\begin{bmatrix} \dot{u}_s \\ \ddot{u}_s \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{k(u_s)}{m} & -\dfrac{1}{m}\mathrm{c}(\dot{\mathrm{u}}_\mathrm{s}, \varphi_i') & 0 \\ 0 & -\dfrac{\varphi_i'}{L} & -\dfrac{R_T}{L} \end{bmatrix} \begin{bmatrix} u_s \\ \dot{u}_s \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ \ddot{x}_b + g \\ 0 \end{bmatrix} \tag{26}$$

Now, the last form of the system of equations is ready to be implemented in the computational code.

It is important to note that the RK method is particularly attractive for solving this class of problems since it can handle nonlinearities without requiring the linearization of the equations of motion. This advantage is in some way paid with the additional degree of freedom required to put the second order equations of motion in the equivalent first order state space form.

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4 RESULTS

In order to validate the proposed approach a physical prototype of a LBVEH was built. The key idea of the physical design is not to optimize the power generation but to obtain a dynamical behavior such that the voltage signal is composed by multiple magnet fluxes crossing different coils simultaneously; this choice is made to test the framework in the most complex scenario. The harvester was designed with a multi-pole-multi-coil configuration that generates a complex magnetic field distribution; thus, during operation the flux gradient curve crosses different coils and a complex voltage signal is generated. The coils were wounded with commercial wire of AWG30 gauge and connected in series.

The stack was built by piling four neodymium magnets in repelling poles configurations; steel spacers were added between the magnets in order to retain the magnetic flux. The stack displacement is limited by the repelling force of two end magnets, which are identical to those that form the stack; this choice allows to generate a large displacement dynamics.

4.1 Testing

The physical prototype was excited with a electromechanical shaker a different frequencies and the voltage and base acceleration signals were acquired. The equipment used to conduct the tests was: Labworks ET-132 electrodynamic shaker, Rigol DG 4062 function generator, home-built amplifier, PCB Accelerometer and National NI9234 data acquisition module.

The harvester was excited at different discrete frequencies between 2 and 10 Hz. The piezoelectric accelerometer was mounted in the base of the shaker in order to use this signal as an excitation for the computational model.

The formulation was implemented in a Python code written by the authors called PyDy, which is based on the Object Oriented Programming philosophy and a Runge-Kutta Method solver.

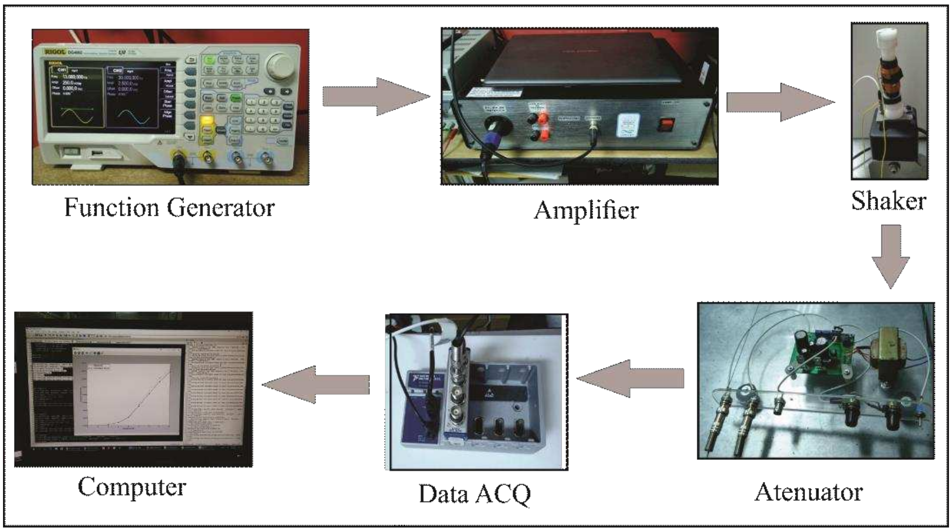

Figure 4 – Data acquisition hardware configuration

4.2 Damping identification

The present approach includes the effects of both viscous and friction damping. The friction damping is expected to influence the system dynamics in the low frequency range; viscous damping is expected to do it in the high frequency range.

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Both damping effects are modeled through proportional models with frequency varying coefficients, which are a function of velocity and its determination require an "identification" procedure. Especially the friction coefficient is very difficult to identify since the normal force is intermittent. This intermittence is very difficult to eliminate without closing the gap between the stack and the cylinder, which has the drawback of increasing the effective friction force and consequently the value of damping. In order to simplify the identification of the damping coefficients, we have assumed that the stack moves at the same frequency of the signal and then identified discrete values of damping for certain frequencies.

The Figure 5 shows the simulated and measured voltage signals at an operation frequency of 2 Hz for different friction coefficients, the viscous damping coefficient was set to 0.25.

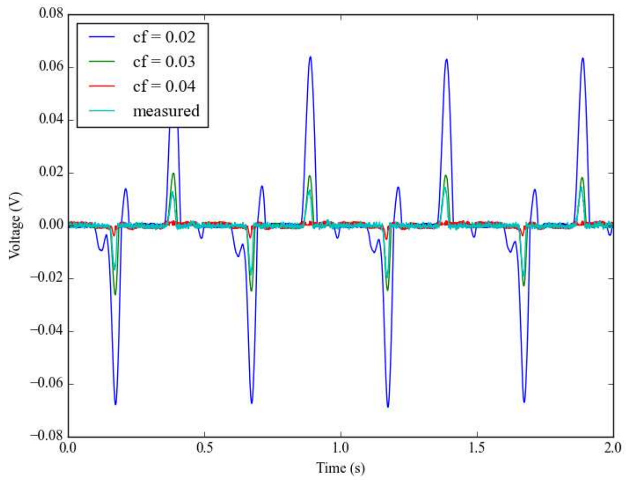

Figure 5 – Effect of the frictional force

It can clearly be seen that friction governs the intermittence of the stack motion; the stack moves only when the inertial force is larger than the frictional force.

The Figure 6 shows the correlation between the experiment and the simulation for one period of motion setting the frictional damping coefficient at 0.33.

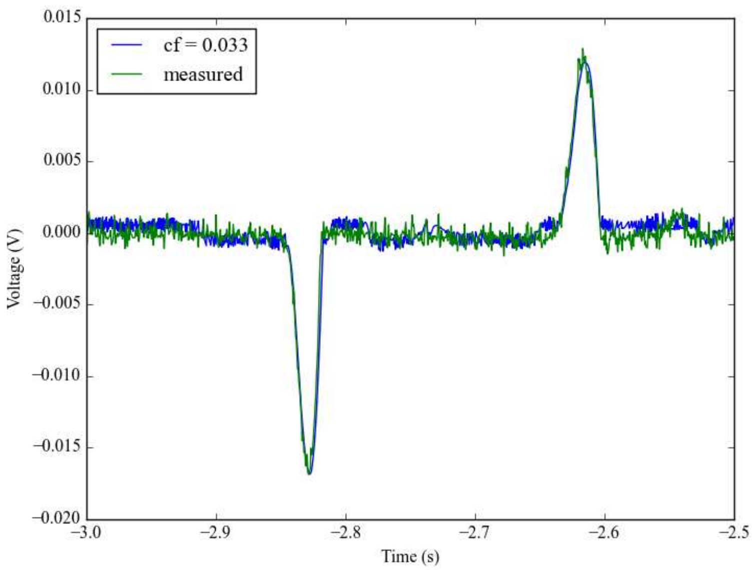

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Figure 6 – Friction coefficient identification – 2.0 Hz

As the excitation frequency increases the effect of the friction damping is less important and the viscous damping terms governs the damping forces. The Figure 7 shows the correlation of the voltage signal between the experiment and the simulation for a base excitation of 10.0 Hz. The optimal viscous damping coefficient is 0.4.

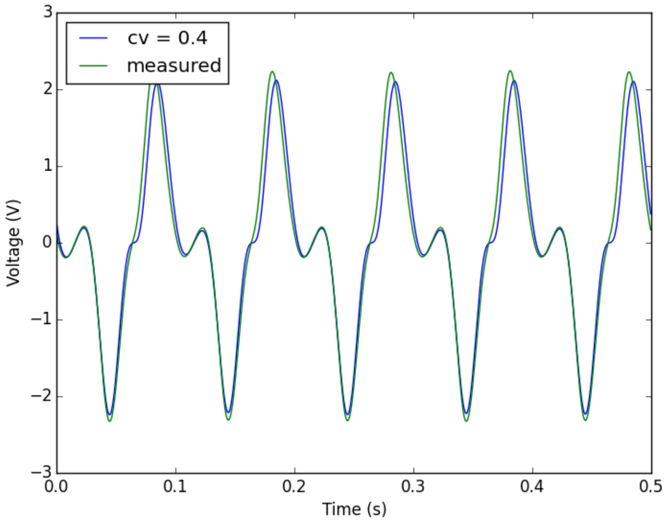

Figure 7 – Friction coefficient identification – 10.0 Hz

Note that obtaining such a good correlation of the voltage signal in the time domain is actually difficult since it is dictated by the product of two functions that are changing in space and time, the velocity and the magnetic flux derivative.

4.3 On the existence of resonance

Most investigations about magnetic levitation based energy harvesters (LBVEH) assume that the harvester must work at resonance to scavenge the maximum energy [6,7,8,11]. However, the harvester behaves as a Duffing oscillator, which do no exhibit resonance (Meirovitch, 1986). Resonance is defined as a property of the system and thus it must be independent of the external forces. But this is not the case of an oscillator with nonlinear stiffness, since the stiffness is a function of the displacement; then it could be said the system has a “continuously varying natural frequency”. Note that the displacement amplitude of an undamped LBVEH increases continuously with frequency, and the maximum amplitude is found when the forcing frequency is infinity.

When the system is damped, the stack amplitude does not increase continuously with the frequency of the excitation; for a certain frequency, the response falls suddenly from a large displacement equilibrium to a small displacement equilibrium. The phenomenon is called “jump” and is typical of the Duffing oscillator. The frequency location of the maximum amplitude is not only a function of the system, but also of the load amplitude and the system damping.

4.4 Generated power

The ultimate objective of the proposed computational procedures is to obtain an accurate estimation of the harvested energy. The electrical power is a function of the induced voltage and the current flowing in the circuit; both variables are related through

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the circuit load. In order to simulate the power consumption of a small electronic device we have represented the load with a resistance of 100 Ohm.

The Figure 8, Figure 9 and Figure 10 show the correlation of the peak voltage for an input acceleration of 1g, the average power and the peak power between the present computational approach and the physical experiment. The tests were performed at discrete values of excitation frequencies in order to avoid transient effects; the base accelerations measured in the physical tests were used as input of the simulation.

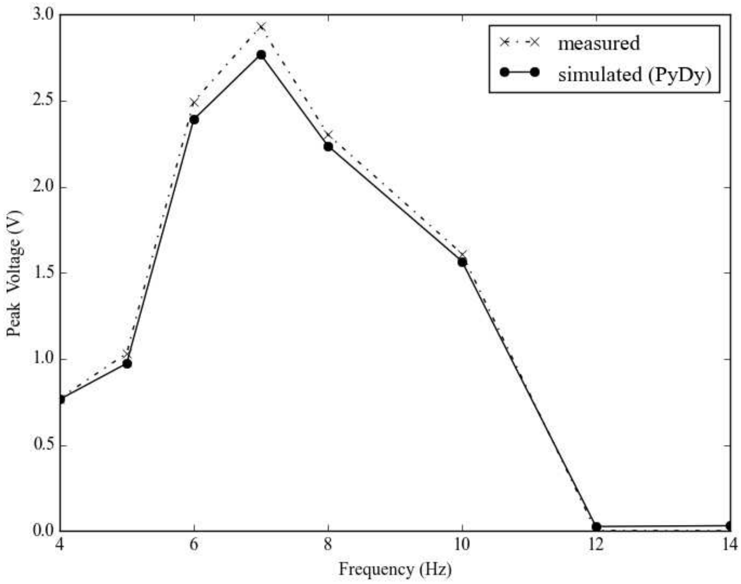

Figure 8 – Peak Voltage (1g)

The peak voltage values given by the present approach correlate very well with the experiment, see Figure 11. It must be noted that in order to obtain accurate values of the peak voltage it must be ensure that the excitation of the computational model is the same than the excitation of the experiment; especially if the shaker dimensions are such that the system inertial forces affects its dynamics.

Regarding the power values, both the peak and the average power show a good agreement between the experiment and the simulation. It is important to mention that the time signals of power are directly obtained from the voltage signal, which also shows an excellent correlation between the simulation and the experiment; see Figure 5, Figure 6 and Figure 7.

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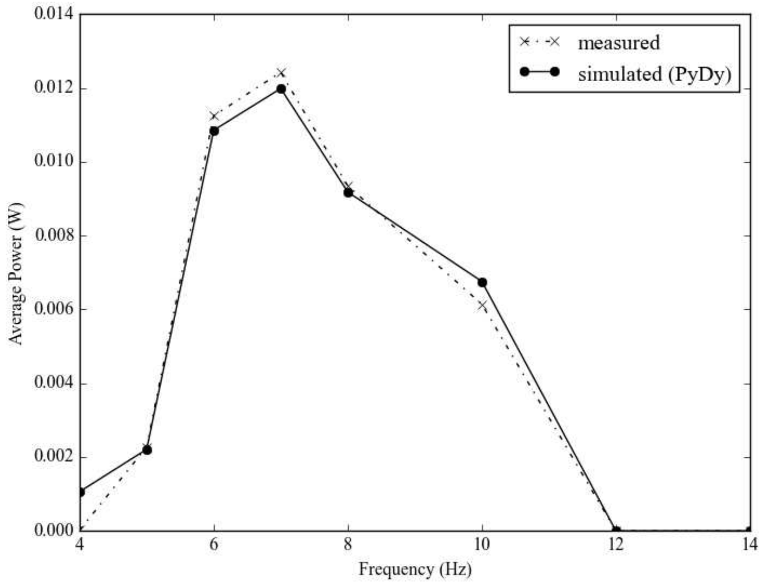

Figure 9 – Average Power (1g)

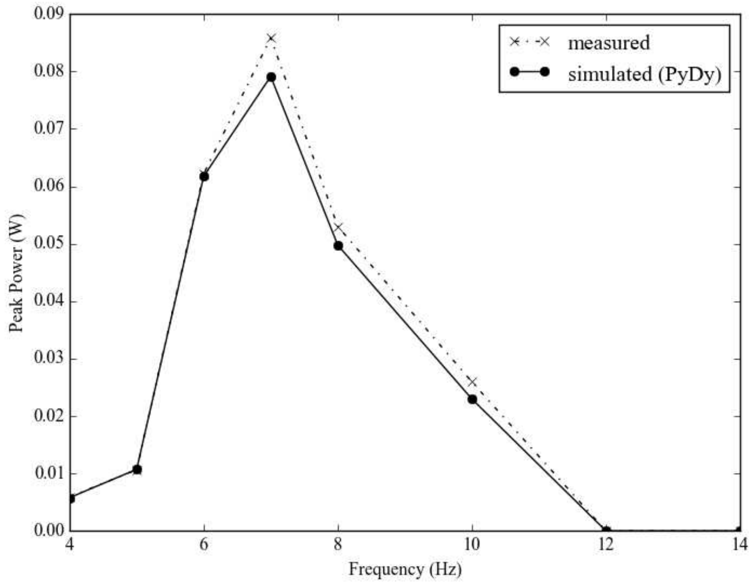

Figure 10 – Peak Power (1g)

5 CONCLUSIONS

A hybrid numerical-analytical approach for the design of levitation based vibration energy harvesters has been presented. The coupled equations of motion of the 2 DOF electromagnetic system were written in terms of an analytical function for the levitation force and a numerically interpolated function for the magnetic flux derivative. The approach can model harvesters with arbitrary magnet-coil-spacer configurations; it allows arbitrary excitations and arbitrary nonlinearities.

The levitation force function was obtained via polynomial fitting of discrete force measurements of a finite element model. It was shown that numerical modeling of the levitation force and the magnetic flux can be embedded into the electromagnetic dynamic equations to permit modeling of any multi-pole-multi-coil configuration. Using the same

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finite element model a numerical function of the average magnetic flux density as a function of the spatial coordinate was extracted. This function was derivate numerically and then smoothed through a Savitzky-Golay filter.

The dynamic response of a multi-magnet-multi-coil harvester was analyzed; it was shown that the maximum displacement frequency is strongly dependent on damping and on the excitation amplitude. It was also shown that the response of the harvesters does not exhibit a behavior analogous to the linear oscillator resonance.

A performance assessment by comparing the results obtained with the present formulation against measurements was presented; a physical prototype of a multi-pole-multi-coil harvester is built ad hoc. It was shown that the approach gives excellent results in terms of: prediction of the voltage-time signal and estimation of integrated parameters as average power and average voltage.

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